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Technical Notes

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Modeling Acceleration Effects on Ram Accelerator Thrust at High Pressure

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Nomenclature

A = tube section
a = acceleration
c_p = isobaric heat capacity
F = thrust
h = specific enthalpy
I = nondimensional thrust, $F/(pA)$
L = length
M = Mach number
m = mass
P = pressure ratio
p = pressure
Q = nondimensional heat release, $\Delta q/c_pT$
R = gas constant
T = temperature
u = velocity
v = specific volume
α = intermediate calculation parameter, $L_{\mathrm{CV}}a_p$
Γ = adiabatic specific heat capacity ratio
γ = specific heat capacity ratio
Δq = heat release
η = calorific imperfection, h/c_pT
σ = compressibility factor

Subscripts

CJ = Chapman–Jouguet state
CV = control volume
P = projectile
1 = state of reactive mixture
2 = state of the products

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Introduction

AN experimental and theoretical research program has been carried out to determine the parameters that allow successful operation of ram accelerator in dense, gaseous propellants. For this purpose, the ram accelerator of the University of Washington has been used and operated at propellant fill pressures up to 20 MPa. Prior analytical studies showed that at moderate accelerations, that is, on the order of 15,000 g, the unsteady terms in the conservation equations for the thermally choked propulsive mode scarcely exceed a few percent of the magnitudes of the steady convective terms, which allowed them to be neglected.[1,2] At the high acceleration levels that typically arise from operating the ram accelerator at elevated pressure, however, the quasi-steady one-dimensional model overpredicts the experimental acceleration when a real-gas equation of state (EOS) is used. Therefore, a revision to the quasi-steady model was made by Bundy et al.[3] to account for projectile acceleration on the thrust, the finite length of the combustion zone, and the dependency of the real-gas heat release on the in-tube Mach number.

The unsteady performance model of Bundy et al. used the ideal-gas EOS, together with a constant heat capacity for the both the reactants and combustion products, and the real-gas heat release–Mach number profile calculated with the quasi-steady model. Because a sensitivity analysis had shown that the ram accelerator thrust was most affected by changes in heat release due to real-gas effects, the influence of the real-gas EOS on the sound speed and pressure at the thermal choking point were ignored in this previous modeling effort. It was found that the Mach number dependence of the nondimensional thrust predicted by the unsteady model deviated considerably from that predicted by the quasi-steady model at high fill pressure, but it agreed more closely with experimental observations. The aim of the present study is to improve the capability of this unsteady one-dimensional modeling by implementing it with more accurate real-gas corrections based on a virial-type EOS[4] and to examine the influence of control volume length–Mach number variations on the thrust characteristics of the thermally choked ram accelerator.

Unsteady One-Dimensional Conservation Equations

A revised unsteady model that includes the effects of a real-gas EOS for the combustion products was developed, which determines the effect of projectile acceleration on the net ram accelerator thrust as a global process between the state of the propellant entering the control volume and the state of the thermally choked exit flow, as shown in Fig. 1. In the reference frame of the projectile, the mass, energy, and momentum conservation equations were applied to the propellant flow entering and leaving the control volume, which has a length L_{CV}. Conditions at the entrance and exit planes are identified

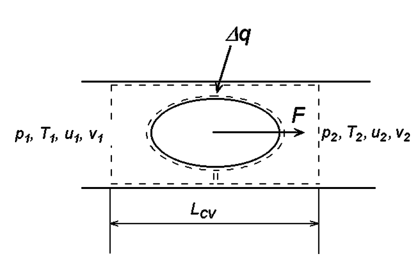

Fig. 1 One-dimensional control volume for ram accelerator process.

by subscripts 1 and 2, respectively. External heat addition and the rate of change of axial momentum are, respectively, characterized by the chemical heat release Δq and net axial force F of the projectile acting on the control volume. (Ram accelerator thrust is equal and opposite to F.)

Analysis of all of the terms in these equations yields a readily applicable set of equations in the form expressed by Bundy et al.[3] After some algebraic manipulation of these relationships, while specifying the end state to be thermally choked, that is, $M_2^2 = \Gamma_2 R_2 T_2 = 1$, and introducing a real-gas EOS,[4,5] namely, $pv/RT = \sigma(v, T)$, the following expressions are derived:

$$\frac{T_2}{T_1} = \frac{c_{p1}}{c_{p2}} \frac{\left(\eta_1 + M_1^2 \gamma_1 R_1 \big/ 2c_{p1} + Q\right) - (7/2)(\alpha / c_{p1} T_1)}{(\eta_2 + \Gamma_2 R_2 / 2c_{p2})\left[1 - \alpha \big/ R_1 M_1^2 \gamma_1 T_1\right]} \tag{1}$$

$$P = \frac{\sigma_2}{\sigma_1}\left[M_1 \sqrt{\frac{\gamma_1 R_2}{\Gamma_2 R_1}} - \frac{\alpha}{R_1 M_1 T_1}\sqrt{\frac{R_2}{R_1 \gamma_1 \Gamma_2}}\right]\sqrt{\frac{T_2}{T_1}} \tag{2}$$

$$I = \frac{\alpha}{R_1 T_1} + \frac{\sigma_2}{\sigma_1}\left(M_1 \sqrt{\frac{\gamma_1 R_2}{\Gamma_2 R_1}} - \frac{\alpha}{R_1 M_1 T_1}\sqrt{\frac{R_2}{R_1 \gamma_1 \Gamma_2}}\right) \times \sqrt{\frac{T_2}{T_1}}\left(1 + \frac{\Gamma_2}{\sigma_2}\right) - \left(1 + M_1^2 \frac{\gamma_1}{\sigma_1}\right) \tag{3}$$

For brevity, the steps required to justify the approximation of the integral terms of the unsteady conservation equations with algebraic expressions that lead to the preceding set are not included here; see the previous study of Bundy et al.[3] for details. As shown in that work, in the limit of infinite mass, the thrust predicted by the unsteady theory is exactly that predicted by the quasi-steady model for thermally choked ram accelerator operation, that is, infinite mass results in no acceleration. For an accelerating projectile in a quiescent propellant, however, the projectile mass is coupled with the acceleration via $I = m_p a_p / pA$, and an iterative approach is required to determine the unique I and a_p that satisfy the governing equations for a given projectile mass and Mach number.

Calculation Procedure

Previous studies showed that the use of a real-gas EOS for the initial state 1 did not influence the results.[6] Therefore, throughout the present study, the ideal-gas EOS is taken to describe the initial properties of the mixture, that is, $\sigma_1 = 1$. For state 2, the value of σ_2 is derived from a virial EOS.[4,5] The thermodynamic properties of the products were primarily determined from the ram accelerator computer code developed at the University of Washington (UW).[7] To account more accurately for real-gas effects, the c_p values at the thermal choking point determined from the UW code were corrected by the more exact calculations using the QUATUOR code[4,5,8] and the virial EOS formulation.[6]

Unlike in the quasi-steady-state assumption, the preceding equations show that the nondimensional thrust I is a direct function of both the length of the control volume and the acceleration. An iterative procedure was used to solve for the value of α in Eq. (3) for an arbitrarily chosen value for L_{CV}. The initial value for a_p was that of the steady-state calculation. After the value for α converged, it was applied in Eqs. (1) and (2) to yield the values of T_2/T_1 and P, respectively. These values were then used to compute a new σ_2, and the iteration process was repeated until the α, σ_2, T_2/T_1, and P terms had all converged. This approach results in a slightly different heat release–Mach number profile than predicted from the quasi-steady model.

Comparison with Experimental Data and Discussion

The parameter L_{CV} is a key element in the unsteady modeling because it appears in all of the governing equations (1–3) via the term α. In experiments at fill pressures of 2.5 MPa, it was observed that the combustion during thermally choked ram accelerator operation at Mach 3.7 is typically completed within approximately one projectile length behind the projectile base.[1] Therefore, the value $L_{CV} = 2L_p$ was chosen in previous studies.[1–3] Recent experiments were carried out at both 15- and 20-MPa fill pressure in a 4-m-long, 38-mm-bore ram accelerator test section, with 118-g titanium alloy projectiles.[3] In the present work, the influence of L_{CV} on the theoretical nondimensional thrust vs Mach number behavior is compared in Fig. 2 with the 15-MPa experiment. Based on signal distortion and sensor sampling rate (1 MHz) of center-differenced time–distance data, the experimental Mach–distance records were determined with a Mach number uncertainty of about $\pm 3\%$. The experimental acceleration was determined by double-differentiating time–distance data from instrumentation stations that are separated by 1.0–1.7 m, which results in uncertainties of $\sim 70\%$ for the nondimensional thrust. These uncertainties are indicated by the horizontal and vertical error bars in Fig. 2.

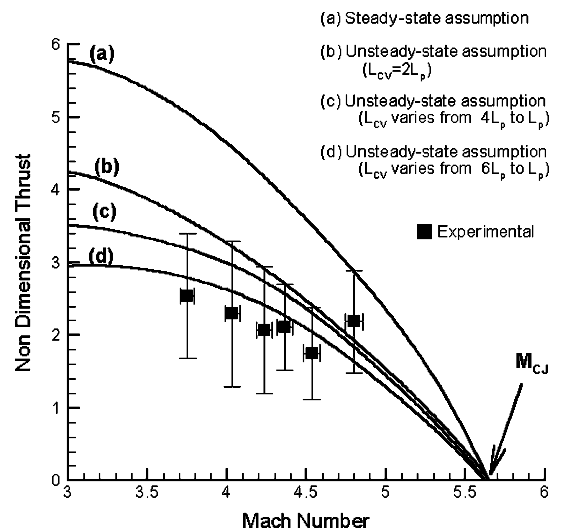

Fig. 2 Experimental thrust–Mach data with unsteady modeling; $2.6CH_4 + 2O_2 + 9.2N_2$ propellant at $p_1 = 15$ MPa, $m_p = 118$ g projectile mass.

One of the theoretical curves in Fig. 2 was calculated with the assumption that L_{CV} was fixed at twice the projectile length, as has been done previously.[1–3] The length of the combustion zone, however, is expected to decrease with increasing Mach number because the static temperature increases, which, in turn, enhances the chemical kinetic rates. To examine the effects of a Mach number-dependent control volume length, the nondimensional thrust vs Mach number behaviors were determined when L_{CV} varied linearly from both $4L_p$ to L_p and $6L_p$ to L_p over the Mach number range of 3 to M_{CJ}. The larger the value of L_{CV} at lower Mach number, the greater the reduction in thrust is from that of the steady-state prediction. All of the thrust coefficient curves converge to zero at the same Mach number because the zero thrust condition for thermally choked flow at a given pressure is always that of a Chapman–Jouguet detonation wave in the propellant. Consequently, the significance of the Mach number dependence for L_{CV} diminishes as the projectile velocity approaches that of M_{CJ}.

The control volume length that would be most appropriate in the unsteady ram accelerator model at a given projectile Mach number is very much related to the chemical kinetics of the process. Although it has been shown that the conical shock wave generated by the projectile nose tip and its multiple reflections before the combustion zone heat the flow in the ram accelerator,[9] only the effect of a single shock wave was examined here to determine the order of magnitude of the change in length of the combustion zone one may expect as the Mach number is increased. The Mach number dependence of induction length behind an incident shock for the $2.6CH_4 + 2O_2 + 9.2N_2$ propellant, used in this study, was calculated with the CHEMKIN computer code[10] using the GRI kinetic scheme[11] for methane combustion under ideal-gas EOS conditions. The results are shown in Fig. 3. Although this kinetic scheme for fuel-rich propellants at high pressure has not been validated, these calculations do provide a qualitative description of the variation of

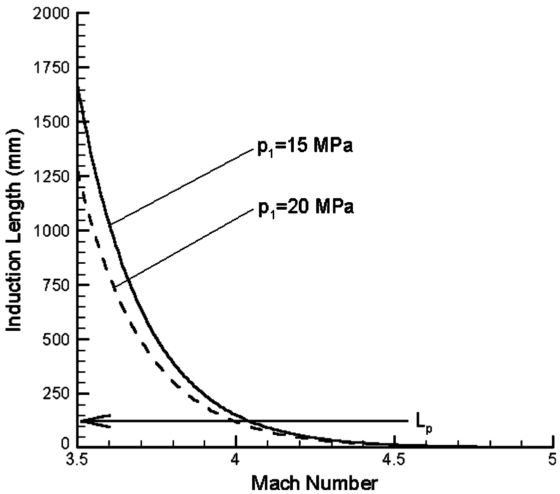

Fig. 3 Variation of calculated induction length downstream of normal shock wave.

L_{CV} with Mach number at velocities below M_{CJ}. Figure 3 shows that, in the Mach number range of 3.7–4, L_{CV} varies exponentially between $6L_p$ and L_p, where L_p is 125 mm. The goal of the present study, however, is not to determine the best fit for the Mach number dependence of L_{CV}, but rather to illustrate that performance predictions are very sensitive to the choice of these models. A more thorough exploration of the character of the dependence of L_{CV} on Mach number is warranted, but has not yet been carried out.

Some of the other factors that significantly influence the control volume length are real-gas reaction kinetics, turbulence, shock–boundary-layer interaction, and heating of the flow by multiple shocks. Consequently, a more refined analysis involving a detailed modeling of the flowfield from the projectile nose tip to the choking point is needed to determine accurately the Mach number dependence of the control volume length. The time variation of L_{CV} may also be an important term in the control volume analysis; however, the impact of this change is much less than that due to projectile acceleration. Future work should attempt to incorporate as many of these factors as possible to better model the combustion zone characteristics of an accelerating, thermally choked ram accelerator projectile.

Summary

This investigation was aimed at demonstrating the pertinent use of a real-gas assessment in the unsteady, one-dimensional modeling of thermally choked ram accelerator thrust, utilizing a virial EOS and a Mach number-dependant combustion zone length. The equilibrium composition of the combustion products at the thermal choking point was iterated until the real-gas acoustic speed, temperature, and pressure completely satisfied the conservation equations of the unsteady model. This results in a slightly different heat release–Mach number profile than predicted from the quasi-steady model and accounts for real-gas effects on the acoustic speed and pressure at the thermal choking point. The nondimensional thrust predicted by the unsteady modeling, which is less than that derived from a quasi-steady-state assumption at all Mach numbers, was found to be highly dependent on the how the control volume length was assumed to vary with Mach number. An inverse linear dependence of control volume length on Mach number turned out to improve significantly the agreement between the theoretical thrust–Mach predictions and experimental data than those determined when assuming constant L_{CV}. Kinetic calculations of the induction zone length behind a normal shock wave with an ideal-gas EOS indicated that the L_{CV} could readily be an order of magnitude longer than the projectile at Mach numbers less than $M = 3.5$. Thus, more accurate modeling of the combustion process utilizing kinetic schemes appropriate for high-pressure combustion needs to be pursued.

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